# Klasifikacija iskaznih formula

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$$A \wedge (B \wedge C) \cong (A \wedge B) \wedge C$$

$$A \wedge B \cong B \wedge A$$

$$A \wedge \top \cong A$$

$$C \to (B \to A) \cong (C \wedge B) \to A$$

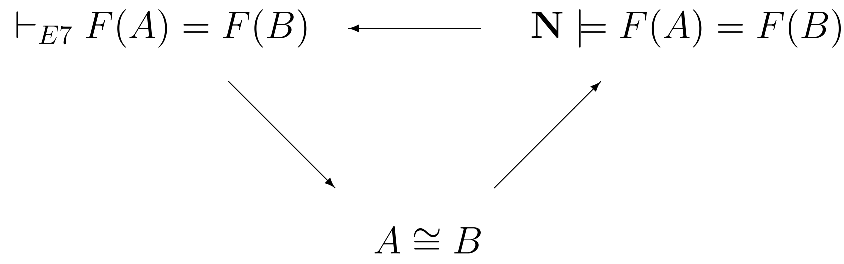
$$\top \to A \cong A$$

$$C \to (A \wedge B) \cong (C \to A) \wedge (C \to B)$$

$$A \to \top \cong \top$$

$E7$

| | |
|---|---|
| $x \cdot (y \cdot z) = (x \cdot y) \cdot z$ | $A \wedge (B \wedge C) \cong (A \wedge B) \wedge C$ |
| $x \cdot y = y \cdot x$ | $A \wedge B \cong B \wedge A$ |
| $x \cdot 1 = x$ | $A \wedge \top \cong A$ |
| $(x^y)^z = x^{y \cdot z}$ | $C \to (B \to A) \cong (C \wedge B) \to A$ |
| $x^1 = x$ | $\top \to A \cong A$ |
| $(x \cdot y)^z = x^z \cdot y^z$ | $C \to (A \wedge B) \cong (C \to A) \wedge (C \to B)$ |
| $1^x = 1$ | $A \to \top \cong \top$ |




S.V. Soloviev, *The category of finite sets and cartesian closed categories* (in Russian), ***Zapiski Nauchnykh Seminarov (LOMI)***, vol. 105 (1981), pp. 174-194 (English translation in ***Journal of Soviet Mathematics***, vol. 22 (1983), pp. 1387-1400)

M. Fiore, R. Di Cosmo and V. Balat, *Remarks on isomorphisms in typed lambda calculi with empty and sum types*, ***Annals of Pure and Applied Logic***, vol. 141 (2006), pp. 35-50

$$A \wedge (B \wedge C) \cong (A \wedge B) \wedge C$$

$$A \wedge B \cong B \wedge A$$

$$A \wedge \top \cong A$$

$$C \to (B \to A) \cong (C \wedge B) \to A$$

$$\top \to A \cong A$$

$E5$

$$x \odot (y \odot z) = (x \odot y) \odot z \qquad A \wedge (B \wedge C) \cong (A \wedge B) \wedge C$$

$$x \odot y = y \odot x \qquad A \wedge B \cong B \wedge A$$

$$x \odot 2 = x \qquad A \wedge \top \cong A$$

$$(x^{\underline{y}})^{\underline{z}} = x^{\underline{y \odot z}} \qquad C \to (B \to A) \cong (C \wedge B) \to A$$

$$x^{\underline{2}} = x \qquad \top \to A \cong A$$

Za diversifikovane formule $A$ i $B$ važi:

$$
\begin{array}{ccc}
\vdash_{E5} G(A) = G(B) & \longleftarrow & \mathbf{N} \models G(A) = G(B) \\
\searrow & & \nearrow \\
& A \cong B &
\end{array}
$$

K.D. and Z.P., *Isomorphic objects in symmetric monoidal closed categories*, ***Mathematical Structures in Computer Science***, vol. 7 (1997), pp. 639-662

$$A \wedge (B \wedge C) \cong (A \wedge B) \wedge C$$

$$A \wedge B \cong B \wedge A$$

$$A \vee (B \vee C) \cong (A \vee B) \vee C$$

$$A \vee B \cong B \vee A$$

$E2{+}2$

$$p \wedge (q \wedge r) = (p \wedge q) \wedge r \qquad A \wedge (B \wedge C) \cong (A \wedge B) \wedge C$$

$$p \wedge q = q \wedge p \qquad A \wedge B \cong B \wedge A$$

$$p \vee (q \vee r) = (p \vee q) \vee r \qquad A \vee (B \vee C) \cong (A \vee B) \vee C$$

$$p \vee q = q \vee p \qquad A \vee B \cong B \vee A$$

Za diversifikovane formule $A$ i $B$ važi:

$$\begin{array}{ccc} \vdash_{E2+2} A = B & \longleftarrow & \models A \leftrightarrow B \\ & \searrow \qquad \nearrow & \\ & A \cong B & \end{array}$$

PROPOSITION. *Za formule $A$ i $B$ takve da se samo iskazna slova pojavljuju u oblasti dejstva $\neg$ u njima, važiće da su $A$ i $B$ izomorfne* akko *je $A \leftrightarrow B$ tautologija i svako slovo koje se pojavljuje pozitivno u $A$ se takodje pojavljuje pozitivno u $B$ isti broj puta i obrnuto, a takodje isto važi i za negativna pojavljivanja slova u $A$ i $B$.*

K.D. and Z.P., *Isomorphic formulae in classical propositional logic*, ***Mathematical Logic Quarterly***, vol. 58 (2012), pp. 5-17

http://www.dicosmo.org/ResearchThemes/ISOS/ISOShomepage.html