

Title: Constructive semigroups with apartness - a new approach to semigroup theory

The theory of constructive semigroups with apartness are a new approach to semigroup theory, and not a new class of semigroups. Of course, our work is partly inspired by classical semigroup theory, but, on the other hand, it is distinguished from it by two significant aspects: first, we use intuitionistic logic rather than classical, secondly, our work is based on the notion of apartness (between elements, elements and sets). In short, framework of our work is constructive mathematics - roughly, mathematics with intuitionistic logic. Constructive mathematics is not unique notion. Principle trends include the following varieties: INT, RUSS, BISH. Constructive mathematics in our work (and in this talk) is Errett Bishop - style constructive mathematics, BISH, [2]. Despite the fact that constructive algebra is (relatively) old discipline (developed among others by L. Kronecker, van der Waerden, A. Heyting), it is, compared with constructive analysis and topology, still of modest size. Following [1], the principal novelty in treating basic algebraic structures constructively is that apartness becomes a fundamental notion, i.e. one axiomatizes rings, groups, and fields with apartness. The main aim of our work within constructive semigroups with apartness (was, is, and) will be to give a little progress in that direction. Although the lecture will be based on material given in [4,5], it is, by no means an attempt to give a complete overview of our existing results. Important source of ideas and notions of our work is [3]. An example of application(s) of these ideas can be found in [6]. The standard reference for constructive algebra is [7].

References:

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